**Definition 1.**
Let $X, Y$ be topological spaces.

$$X * Y := X \times Y \times [0,1]/(\{X \times \{y\} \times \{1\} | y \in Y\} \cup \{\{x\} \times Y \times \{0\} | x \in x\})$$

is called the join of topological spaces.

This gives rise to a simple definition of a cone.

**Definition 2.**
Let $X$ be topological space. The cone of $X$ can be defined via $CX := \Delta^0_{top} * X$.

**Definition 3.**
Let $G = (V_G, E_G)$, $H = (V_H, E_H)$ be directed graphs.
The join of $G$ and $H$ is defined via

$$G * H := (V_G \cup V_H, E_G \cup E_H \cup V_G \times V_H).$$

**Definition 4.**
Let $G = (V, E)$ be directed graph, $v \notin V$. We define $(\{v\}, \emptyset) * G$, resp. $G * (\{v\}, \emptyset)$, to be the right, resp. left, cone of $G$.

**Definition 5.**
Let $M_{pos} = (M, \geq_M)$, $N_{pos} = (N, \geq_N)$ be partially ordered sets and $\geq$ the transitive relation on $M \cup N$ generated by $a \geq b \Leftrightarrow a \geq_N b \vee a \geq_M b \vee a \in N \wedge b \in M$.
We define $M_{pos} * N_{pos} := (M \cup N, \geq)$ to be the join of $M_{pos}$ and $N_{pos}$.

**Definition 6.**
Let $M_{pos} = (M, \geq)$ be a partially ordered set, $p \notin M$. We define $M_{pos} * (p, \mathrm{eq}_p)$, resp. $(p, \mathrm{eq}_p) * M_{pos}$, to be the right, resp. left, cone of $M_{pos}$.

**Remark 7.**
The join behaves well with the nerve operation, namely we obtain for $X, Y$ directed graphs, partially ordered sets or ordenary categories that $NX * NY = N(X * Y)$.

**Definition 8.**
Let $X, Y$ be simplicial sets. We define $X * Y$ via the n-simplicies

$$(X * Y)_n := \bigcup_{\substack{i,j \geq -1 \\ i+j=n-1}} X_i \times Y_j = X_n \cup Y_n \cup \bigcup_{\substack{i,j \geq 0 \\ i+j=n-1}} X_i \times Y_j$$

where the face and degeneracy maps are defined componentwise.

The representation of $X_n$ and $Y_n$ inside the join give rise to natural inclusions.

**Remark 9.**
Let $X, Y$ be simplicial sets. We obtain $|X * Y| = |X| * |Y|$.

**Definition 10.**
Let $X$ be simplicial set. We define $(\Delta^0 * X)$, resp. $(X * \Delta^0)$, to be the right, resp. left, cone of $X$.

**Beispiel 11.**
Let $i, j \in \mathbb{N}$. The we have $\Delta^i * \Delta^j \simeq \Delta^{i+j+1}$ via the canonical morphism

$$(f, g) \mapsto (x \mapsto \begin{cases} f(x), x < i \\ g(x), \text{ otherwise} \end{cases})$$

**Proposition 12.**
Let $X, Y$ be $\infty$-categories. Then $X * Y$ is an $\infty$-category.

**Notation 13.**
Let $X, Y, Z$ be simplicial sets, $p : X \to Y$ simplicial map.

$$Hom_p(Z * X, y) := \{f \in Hom_{Set_\Delta}(Z * X, Y) | f|_X = p\}.$$

**Definition 14.**
Let $X, Y$ be simplicial sets, $p : X \to Y$ simplicial map. We define the overcategory of p via the universal property: For every simplicial set $Z$ we obtain the equality

$$Hom_{Set_\Delta}(Z, Y/p) = Hom_p(Z * X, Y)$$

**Proposition 15.**
We explicitly express $Y/p$ via it's n-simplices $(Y/p)_n := Hom_p(\Delta^n * X, Y)$.

**Remark 16.**
Set $Y$ be an $\infty$-category, $p$ simplicial map into $Y$. Then $Y/p$ is an $\infty$-category as well.

**Beispiel 17.**
Let $Y$ be topological space and $p : \Delta^0_{top} \to Y$.

$$(Sing(Y)/Sing(p))_n = \{f \in Hom(\Delta^{n+1}_{top}, Y) | f(\chi_{n+1}) \in p(\Delta^0_{top})\}$$

**Beispiel 18.**
Let $X, Y$ be partially ordered sets and $p : X \to Y$ an orderpreserving map.

$$(NY/Np)_n = \{(x_i)_{i\in[n]} | \, \forall \, 0 \leq i \leq j \leq n, x \in X : x_i \leq x_j \leq p(X)\}$$