

The goal is to understand the definition of a stable monoidal  $\infty$ -category enough to understand the universal property of the derived category of abelian groups (or vector spaces for some field), and potentially prove the Eilenberg-Steenrod and de Rham theorems.

1. An  $\infty$ -category is *pointed* if it admits an object which is both initial and final.
2. An  $\infty$ -category is *stable* if it is pointed, finitely complete, finitely cocomplete, and the suspension functor is an equivalence.

Note that one needs the notion of limits to make these definitions.

1. A *colimit* of a diagram  $p : K \rightarrow C$  is an initial object in the cocone category  $C_{p/}$ .
2. An object is *initial* if the canonical map  $C_{/x} \rightarrow C$  is an acyclic fibration of simplicial sets.
3. Cocone categories  $C_{/p}$  are defined via a universal property involving *joins*, an operation on simplicial sets.

1. A *monoidal  $\infty$ -category* is a cocartesian fibration  $p : M^\otimes \rightarrow N(\Delta^{op})$ , such that the Segal maps are equivalences,

$$M_{[n]}^\otimes \rightarrow (M_{[1]}^\otimes)^{\times n}, \quad n \geq 0.$$

2. Let  $p : C \rightarrow D$  be a functor between  $\infty$ -categories. A morphism  $f : c_1 \rightarrow c_2$  in  $C$  is cocartesian if the following map is an acyclic Kan fibration.

$$C_{f/} \rightarrow C_{c_1/} \times_{D_{p(c_1)/}} D_{p(f)/}.$$

3. A functor  $p : C \rightarrow D$  between  $\infty$ -categories is a *cocartesian fibration* if  $p$  is an *inner fibration* and for every object  $c_1 \in C$  and every morphism  $\alpha : p(c_1) = d_1 \rightarrow d_2$  in  $D$  admits a cocartesian lift.