The ∞-category Top of topological spaces is the following simplicial set. An n-simplex is:

(1) A tuple $(X_0, \ldots, X_n)$ of $n+1$ topological spaces.
(2) An tuple of morphisms

$$(h_{i,j} : X_i \times \square_{top}^{j-i-1} \to X_j)_{0 \le i < j \le n}$$

where $\square_{top}^m = \{(t_1, \ldots, t_m) \in \mathbb{R}^m : 0 \le t_i \le 1\}$.
(3) The morphisms $h_{i,j}$ are required to satisfy the compatibility condition: For every $0 \le i < j < k \le n$, we should have

$$\begin{aligned} & h_{i,k}(x, (s_1, \ldots, s_{j-i-1}, 1, t_1, \ldots, t_{k-j-1})) \\ = & h_{j,k}(h_{i,j}(x, (s_1, \ldots, s_{j-i-1})), (t_1, \ldots, t_{k-j-1})) \end{aligned}$$

for all $x \in X_i$, $(s_1, \ldots, s_{j-i-1}) \in \square_{top}^{j-i-1}$, $(t_1, \ldots, t_{k-j-1}) \in \square_{top}^{k-j-1}$.

Notice that a tuple $((X_0, \ldots, X_n), (h_{ij})_{0 \le i < j \le n})$ defines a morphism

$$f_{ij}(-) \stackrel{def}{=} h_{ij}(-, (0, 0, \ldots, 0)) : X_i \to X_j$$

for each $0 \le i < j \le n$. Moreover, for every $i < i_1 < i_2 < \cdots < i_k < j$ the compatibility conditions imply that

$$h_{ij}(-, e_{i_1} + \cdots + e_{j_k}) = f_{i_k,j} \circ f_{i_{k-1},i_k} \circ \cdots \circ f_{i,i_1} : X_i \to X_j$$

where $e_{i'} = (0, \ldots, 0, 1, 0, \ldots, 0)$ is the i'th standard basis vector of $\mathbb{R}^{j-i}$. So we can interpret h_{ij} as a homotopy between all the possible compositions of the f's with f_{ij} at the "lowest" corner of $\square_{top}^{j-i-1}$ and $f_{j-1,j} \circ f_{j-2,j-1} \circ f_{i+1,i+2} \circ f_{i,i+1}$ at the "highest" corner. The compatibility conditions then can be interpreted as asking that these homotopies are compatible with all compositions.

The face morphisms are

$$d_k : (X_0, \ldots, X_n, h_{i,j}) \mapsto (X_0, \ldots, X_{k-1}, X_{k+1}, \ldots, X_n, h'_{i,j})$$

where

$$h'_{i,j}(x,t) = \begin{cases} h_{i,j}(x,t) & i < j < k \\ h_{i,j+1}(x, (t_1, \ldots, t_{k-i-1}, 0, t_{k-i}, \ldots, t_{j-i-1})) & i < k \le j \\ h_{i+1,j+1}(x,t) & k \le i < j. \end{cases}$$

The degeneracy morphisms are

$$d_k : (X_0, \ldots, X_n, h_{i,j}) \mapsto (X_0, \ldots, X_k, X_k, \ldots, X_n, h'_{i,j})$$

where

$$h'_{i,j}(x,t) = \begin{cases} h_{i,j}(x,t) & i < j \le k \\ h_{i,j-1}(x, (t_1, \ldots, t_{k-i-1}, t_{k-i+1}, \ldots, t_{j-i-1})) & i \le k < j \\ h_{i-1,j-1}(x,t) & k < i < j. \end{cases}$$

Here, we interpret $h_{i,i}$ as id_{X_i}.

Note that every sequence of continuous homomorphisms $X_0 \xrightarrow{f_1} \cdots \xrightarrow{f_n} X_n$ defines an n-simplex: choose $h_{i,j}$ to be the composition $X_i \times \square^{j-i-1} \to X_i \xrightarrow{f_{i+1}} X_{i+1} \xrightarrow{f_{i+2}} \cdots \xrightarrow{f_j} X_j$ (i.e., the trivial homotopy).

We can write this data in an upper triangular matrix

$$\begin{pmatrix} X_0 & h_{01} & h_{02} & h_{03} & h_{04} & h_{05} \\ & X_1 & h_{01} & h_{13} & h_{14} & h_{15} \\ & & X_2 & h_{23} & h_{24} & h_{25} \\ & & & X_3 & h_{34} & h_{35} \\ & & & & X_4 & h_{45} \\ & & & & & X_5 \end{pmatrix}$$

Now we will be concerned with morphisms $\Delta^1 * \partial\Delta^n \to Top$. There is a canonical inclusion $\Delta^1 * \partial\Delta^n \subseteq \Delta^{n+2}$, as

$$\Delta^1 * \partial\Delta^n = \cup_{i=2}^{n+2} d_i \Delta^{n+2}.$$

Consequently, a morphism as above corresponds to similar data $((X_i), (h_{ij}))$ and compatibilities as for a morphism $\Delta^{n+2} \to Top$, except, $h_{0,n+2}, h_{1,n+2}, h_{2,n+2}$ have as sources

$$h_{2,n+2} : X_2 \times \partial\square^n \to X_{n+2}$$

$$h_{1,n+2} : X_1 \times \sqcap_{1,1}^{n+1} \to X_{n+2}$$

$$h_{0,n+2} : X_0 \times (\sqcap_{1,1}^{n+1} \cap \sqcap_{1,2}^{n+1}) \to X_{n+2}$$

here we define

$$\begin{aligned} \sqcap_{\epsilon,i}^n &= \{(t_1, \ldots, t_n) : t_j = 0 \text{ or } 1 \text{ for some } j \neq i \text{ or } t_i = 1 - \epsilon\} \\ &= \cup_{(\epsilon',i') \neq (\epsilon,i)} d_{\epsilon',i'} \square^n \end{aligned}$$

Our interest in these morphisms is to calculate the colimit of a morphism $A \to B$.

Actually we are more interested in morphisms $(0 \rightrightarrows 1) * \Delta^n \to Top$ so we can calculate coequalisers. Notice that $(0 \rightrightarrows 1) = \Delta^1 \sqcup_{\partial\Delta^1} \Delta^1$. Since join commutes with colimits we deduce that $(0 \rightrightarrows 1) * \Delta^n = \Delta^{n+1} \sqcup_{\partial\Delta^1 * \Delta^n} \Delta^{n+1}$.

We also have $\partial\Delta^1 * \Delta^n \subseteq \Delta^{n+1}$, as a partially ordered set, is the set of subsets $I \subseteq [n+1]$ such that $\{0,1\} \not\subseteq I$. On the other hand, Λ_i^{n+1} as a partially ordered set is the set of $I \subseteq [n+1]$ such that $I \neq [n+1]$ and $I \neq \{0, \ldots, \hat{i}, \ldots, n+1\}$.

On the other hand, if we use pushouts, then our diagram categories are all 0-categories. The indexing category for a pushout is Λ_0^2. The category $\Lambda_0^2 * \Delta^n$ can be described as the partially ordered set $\{00, 01, 10, 22, 33, 44, \ldots, nn\}$. A morphism $\Lambda_0^2 * \Delta^n \to Top$ is the data of

(1) Spaces $X_0, X_1, X_{1'}, X_2, \ldots, X_n$,
(2) morphisms
   - $X_0 \times \Lambda\square_{top}^{j-1} \to X_j$ for $2 \leq j$
   - $X_i \times \square_{top}^{j-i-1} \to X_j$ for $(i,j) = (0,1), (0,1')$ and all $i < j \in \{1, 1', 2, 3, \ldots, n\}$.
(3) compatibilities as above.

Here,

$$\Lambda\square_{top}^n = \square_{top}^n \amalg_{d_{01}\square_{top}^n} \square_{top}^n = |\Lambda_0^2| \times \square_{top}^{n-1}$$

We can organise this data into an upper triangular matrix

$$\begin{pmatrix} X_0 & h_{01}, h_{01'} & h_{02} & h_{03} & h_{04} & h_{05} \\ & X_1, X_{1'} & h_{12}, h_{1'2} & h_{13}, h_{1'3} & h_{14}, h_{1'4} & h_{15}, h_{1'5} \\ & & X_2 & h_{23} & h_{24} & h_{25} \\ & & & X_3 & h_{34} & h_{35} \\ & & & & X_4 & h_{45} \\ & & & & & X_5 \end{pmatrix}$$